

The Melbourne University Mathematics and Statistics Society

presents

2005 University Maths Olympics

The Questions

These are the questions, as presented to the competitors at the 2005 MUMS University Maths Olympics on the 14th of September 2005. The official answers, as used in the competition are given on the final page. There are some minor flaws in some of the questions. The solutions document [umo05solutions.pdf](#) will present solutions and a commentary on each of the questions and will address some of these flaws.

Questions compiled by Daniel Yeow, Damjan Vukcevic and Joanna Cheng.

Question 1

10 points, 491 points remain

What is the minimum number of times a pen must be lifted off the page in order to draw the following diagram?



Question 2

10 points, 481 points remain

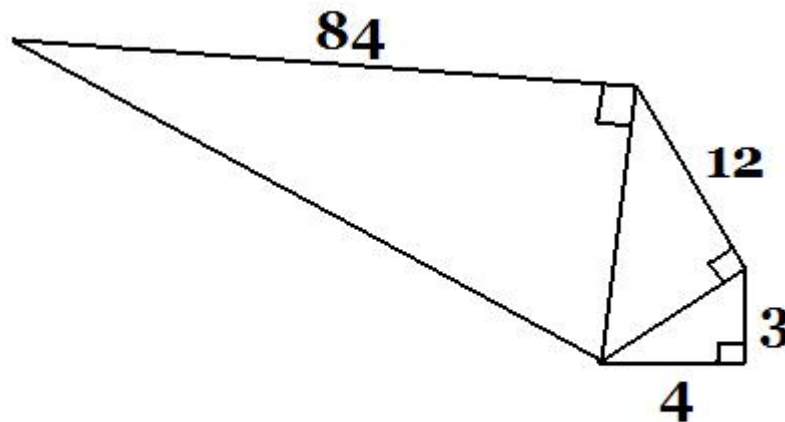
All moons are goons, but only some goons are spoons. At least one moon is little but no spoons are little. If all spoons dislike the goons who are little, and some goons who are not moons are little but dislike spoons...

How many instances of double letters appear on this piece of paper?

Question 3

10 points, 471 points remain

What is the length of the hypotenuse of the largest triangle?



Question 4

10 points, 461 points remain

Andrew is only useless at the Maths Olympics on any day whose name, when spelt in English, contains the letter 't' in it. (forget for the moment about yesterday, today and tomorrow). Nick is only useless on days with an 'i' in them. Geoff is only useless on a Sunday, James is only useless on Monday and Daniel is useless on all the days ending with 'y'. Sally is useless on all the days that Daniel is useless on but on which no-one else is useless on. On what day is Sally useless at the Maths Olympics?

Question 5 **CHANGE RUNNER NOW!** **10 points, 451 points remain**

In a rabbit race, the rabbit who came three places in front of the rabbit who finished last came two places ahead of the rabbit who came seventh. How many rabbits were in the race?

Question 6 **15 points, 436 points remain**

How many powers of 37 are in the number $1^1 \times 2^2 \times \dots \times 2005^{2005}$?

Question 7**15 points, 421 points remain**

How many squares are there on a chessboard?

Question 8**15 points, 406 points remain**

A circular island of radius 100m sits in the exact centre of a regular hexagonal lake of side length 200m. The Queen of the island, being somewhat strapped for cash after purchasing an expensive map of France, needs to build a bridge because she can't swim. What is the minimum length of this bridge, in metres?

Question 9**15 points, 391 points remain**

The year 2005 has reflective symmetry when viewed in digital digits. How many years ago was the last year to have this property?

Question 10 CHANGE RUNNER NOW!**15 points, 376 points remain**

'Twas last Bank Holiday, so I've been told,
Some cyclists rode abroad in glorious weather.
Resting at noon within a tavern old,
They all agreed to have a feast together.
'Put it all in one bill, mine host,' they said,
'For every man an equal share will pay.'
The bill was promptly on the table laid,
And four pounds was the reckoning that day.
But, sad to state, when they prepared to square,
'Twas found that two had sneaked outside and fled.
So, for two shillings more than his due share
Each honest man who had remained was bled.
They settled later with those rogues, no doubt.
How many were they when they first set out?
(note: there are 20 shillings in a pound)

Question 11

20 points, 356 points remain

What is the next number in the following sequence?

1, 4, 9, 61, 52, ...

Question 12

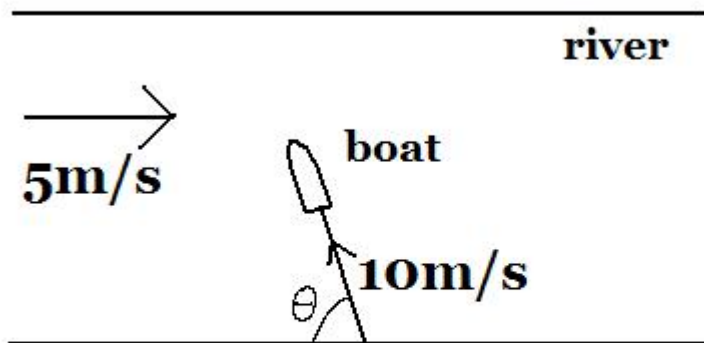
20 points, 336 points remain

The game of Go is played on a 18×18 board composed of rectangles which are $12\text{mm} \times 13\text{mm}$. How many squares are there on a Go board?

Question 13

20 points, 316 points remain

A river with perfectly straight and parallel banks flows west to east at 5 metres per second. You have a badly-drawn boat which can only travel in straight lines and can go exactly 10 metres per second. In order to reach the north bank in the shortest amount of time, what is the angle in degrees, relative to the south bank that must the boat take (see diagram)?



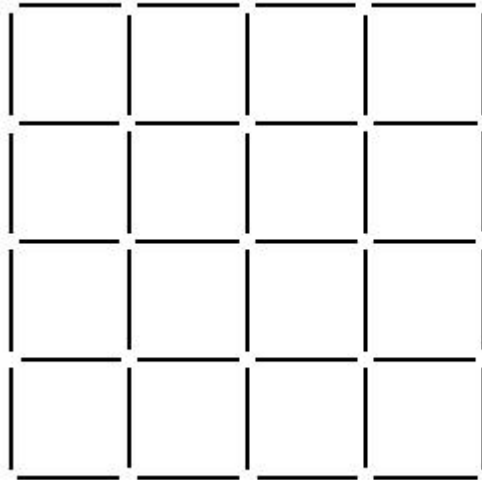
Question 14

20 points, 296 points remain

An equilateral triangle of side length n is divided into n^2 equilateral triangles of side length 1 by lines parallel to its sides, thus giving a network of nodes connected by line segments of length 1. What is the maximum number of segments that can be chosen so that no three chosen segments form a triangle?

Question 15 **CHANGE RUNNER NOW!** **20 points, 276 points remain**

Given a 4×4 square grid made up of matchsticks (i.e. 40 matchsticks), what is the minimum number of matchsticks you can remove such that there are no squares left?



Question 16 **25 points, 251 points remain**

Given a regular tetrahedron (a pyramid whose faces are all equilateral triangles) of side length 2, how far from its centre is one of its vertices?

Question 17**25 points, 226 points remain**

An equilateral triangle with side-length $22\sqrt{3}$ contains two circles which are tangent to each other and are each tangent to two sides of the triangle. The radii of these circles differs by 2. What is the sum of their radii?

Question 18**25 points, 201 points remain**

How many ordered 4-tuples (a, b, c, d) are there such that $a + b + c + d = 25$?
(note: $a + b + c + d = 25$ and $a + b + d + c = 25$ where $c \neq d$ are *different* ways and $a, b, c, d \in \mathbb{N}$).

Question 19**25 points, 176 points remain**

A number n has sum of digits 100, whilst $44n$ has sum of digits 800. Find the sum of the digits of $3n$.

Question 20**CHANGE RUNNER NOW!****25 points, 151 points remain**

$1! = 1$, $2! = 2$, $4! + 0! + 5! + 8! + 5! = 40585$. Find all other numbers that possess this property.

Question 21**30 points, 121 points remain**

ABC is a triangle with area 1. AH is an altitude, M is the midpoint of BC and K is the point where the angle bisector at A meets the segment BC . The area of the triangle AHM is $\frac{1}{4}$ and the area of AKM is $1 - \frac{\sqrt{3}}{2}$. Find the angles of the triangle in degrees.

Question 22

...into the home stretch

30 points, 91 points remain

Find all solutions (x, y) in positive integers to $x^3 - y^3 = xy + 61$.

Question 23 **you can see the finish line...** **30 points, 61 points remain**

What is the minimum real value of $|\sin(x) + \cos(x) + \tan(x) + \cot(x) + \sec(x) + \operatorname{cosec}(x)|$?

Question 24 **Almost there!** **30 points, 31 points remain**

There are 15 sea urchins sitting in 3 rows of 5. The sea urchins leave one at a time. All leaving orders are equally likely. Find the probability that there are never two rows where the number of sea urchins remaining differs by 2 or more.

Question 25

Final Question!

31 points, 0 points remain

Given positive reals a, b, c find all real solutions (x, y, z) to the equations $ax + by = (x - y)^2$, $by + cz = (y - z)^2$, $cz + ax = (z - x)^2$.

Answers

1. 3
2. 19
3. 85
4. Wednesday
5. 8
6. 56314
7. 204
8. $100\sqrt{3} - 100$
9. 124
10. 10
11. 63
12. 42
13. 90
14. $n(n + 1)$
15. 9
16. $\frac{\sqrt{3}}{\sqrt{2}}$, other forms were also accepted
17. 14
18. 2024
19. 300
20. 145
21. 30, 60, 90 in any order
22. (6,5)
23. $2\sqrt{2} - 1$
24. $\frac{72}{7007}$
25. $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ in any order