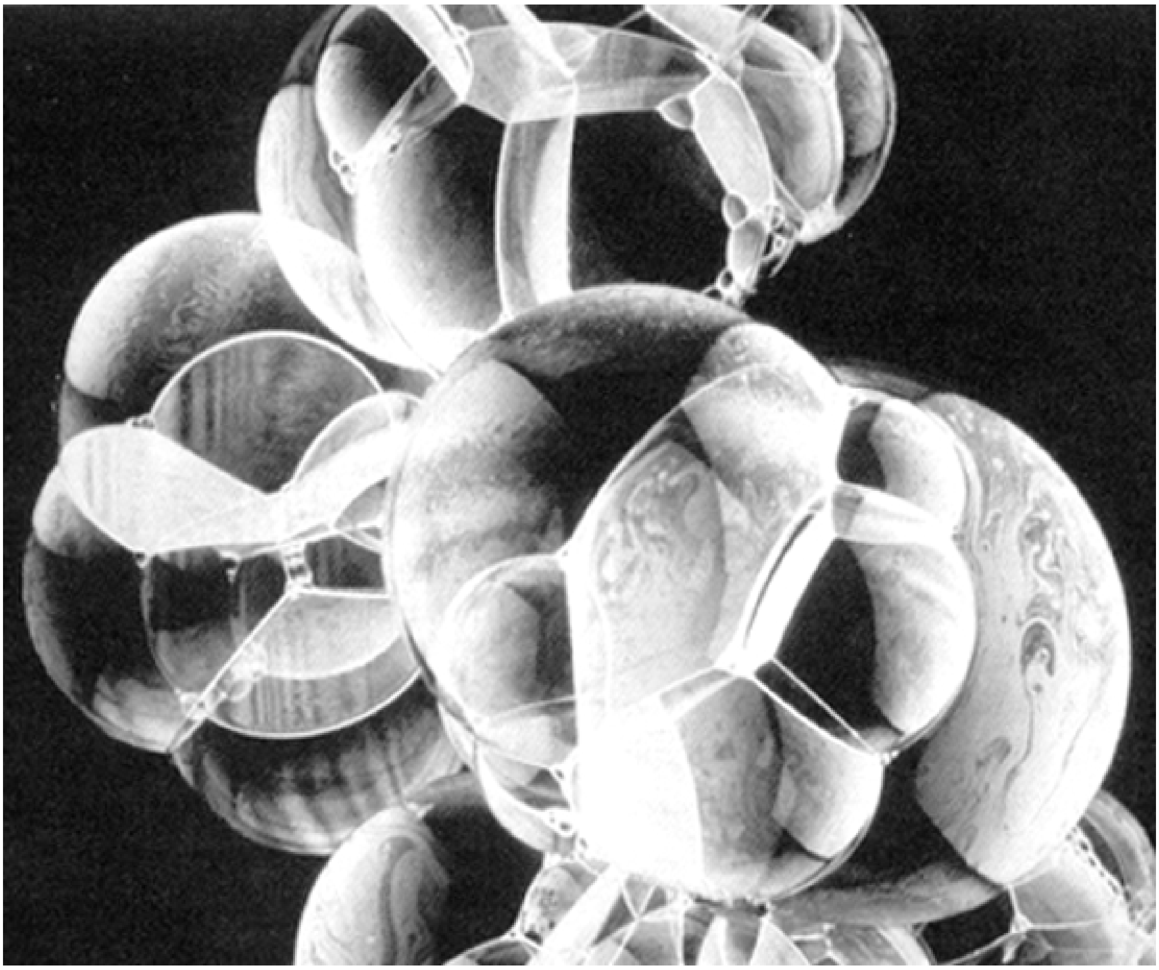
Issue 2, 2000

Paradox

The Magazine of the Melbourne University Mathematics and Statistics Society



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Paradox

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Words from the Editor ...

I'm going to save you some sorrow. Don't flip to the centerfold expecting a Paradox Kid. You won't find one. Paradox Kid is taking a break. Don't worry though, since P.K. and the gang are going to be in the next issue (due early Semester 2) in a bumper double edition. To help dissipate the chaos that you'll feel in his absence, we've provided a chaos simulution game that we hope you'll enjoy. Also in this issue we have articles about an interesting paradox, a big number, and careers in maths.

So far this year MUMS has been very active — congratulations to Des and the new committee for doing a great job. More about that in Norman's article. Remember to check out the noticeboard and website regularly to find out about upcoming MUMS events. Some ones to watch out for are the trivia night — always a big hit, the weekly seminars, and the Friday afternoon tea.

As always, we are very eager to hear from our readers. Either send us an email (`paradox@ms.unimelb.edu.au`) or come and speak to me, or to one of the sub-editors. We'd love to hear from you, and have lots of ideas for articles that you could help us out by writing.

— George Doukas, *Paradox* Editor

About the MUMS barbecue

The date was 4 April 2000, the place, Old Geology courtyard, and the word of the day was barbecue. At about 1:00 p.m. on this fateful Tuesday, the Melbourne University Mathematics and Statistics Society, or MUMS, as it is more affectionately recognised, staged its first social event of the calendar year: a good old-fashioned free sausage sizzle. And not long after the sizzling began, the delicious aroma of MUMS's cooking attracted quite a crowd. Although primarily aimed at first-year maths students, there were people from all faculties, all years, and all walks of life — all united in the celebration of the sausage. (Though vegie burgers were also available for those whose tastes lay elsewhere.)

The hundreds of barbecue enthusiasts who came indulged in an orgiastic feast at the palace, and MUMS chefs worked up a sweat to maintain sufficient sausage output. Even the 1:40 p.m. bread shortage was not enough to hold back the multitude of maths students. A couple of hours, ten slabs of drink, and over 15 kg of meat and meat-substitute later, it appeared that the event had been a great success. Of course, this would not have been possible without the organisational skills of Desmond, the barbecue-tongs dexterity of Thai, and the help provided by the MUMS committee.

— Norman Do

Who wants to be a millionare?

Eddie McGuire has a new game show. Like 'Who wants to be a millionaire', it offers contestants a chance to walk away with a million dollars (tax-free, of course), or perhaps even more. Furthermore, contestants aren't required to answer multiple, trivial questions — they must make a simple choice: one box or two?

The premise is this: Eddie has two boxes. One box is made of clear plastic, and can be seen to contain one thousand dollars in cold, hard cash. The other is opaque, and either contains a million dollars, or nothing. The contestant must choose whether they wish to take both boxes, or just the opaque box. At this point, the choice seems obvious: she should take both boxes and run. There is, however, just one complication. The evil geniuses at GTV9 have come up with a machine that can predict which choice a contestant will make. Before the show, the machine makes its prediction, and Eddie fills the boxes with cash accordingly. If the machine predicts that the contestant will take both boxes, Eddie leaves the opaque box empty. If it predicts she will take just the opaque box, Eddie stuffs the box with a million dollars in cash. During an extensive testing period carried out by the aforementioned evil geniuses, the machine was shown to be 95% accurate in predicting what choice a contestant will make. That is, during testing, whenever a contestant chose just one box, 95% of the time it contained a million dollars. When a contestant chose both boxes, 95% of the time the opaque box was empty and the contestant left with just the thousand dollars (the clear box contains a thousand dollars no matter what the prediction).

What should a contestant on this game show do? What would *you* do? Many people see the obvious answer right away. The problem is, many of those people think it is obvious that you take just the one, opaque box, and many others are certain you should take both. In actual fact, rational arguments can be made for *both* decisions.

You should take both boxes, the argument goes, because no matter what Eddie has actually put in the boxes, you'll still be better off if you take both. If the million is not there, you're better off taking both boxes and getting a thousand dollars than taking just the one and getting nothing. If the million is in the opaque box, then taking both boxes is still better: you'll have a million and a thousand dollars rather than just a million. The situation could perhaps be viewed like this:

	\$million is present	\$million is absent
Take one box	\$1000000	\$0
Take two boxes	\$1001000	\$1000

Looking at the columns, it is clearly better to take two boxes no matter whether the million dollars is in the opaque box or not, therefore you should take both boxes.

However, say one-boxers, this argument ignores the *probability* that the million dollars will be in the opaque box. The predictor is 95% accurate, and so this should be taken into account in any calculations. Thus, to a one-boxer, the issue is not whether the million dollars is present of not, but whether the predictor is correct (which has a probability of 0.95). Adding up the amount of money won multiplied by the probability of each state of affairs for each choice gives the *expected utility*:

	Predictor is correct (0.95)	Predictor is incorrect (0.05)	Expected utility
Take one box	\$1,000,000 x 0.95 = \$950,000	\$0 x 0.05 = \$0	\$950,000
Take two boxes	\$1,000 x 0.95 = \$950	\$1,001,000 x 0.05 = \$50,050	\$51,000

It is ultimately more profitable, taking into account probabilities, if you take just one box. Therefore, you should take just the one box and 95% of the time you will pocket the million dollars. This type of reasoning shows that, even if the predictor is only 51% accurate, it is still more profitable to take just one box:

	Predictor is correct (0.51)	Predictor is incorrect (0.49)	Expected utility
Take one box	\$1,000,000 x 0.51 = \$510,000	\$0 x 0.49 = \$0	\$510,000
Take two boxes	\$1,000 x 0.51 = \$510	\$1,001,000 x 0.49 = \$490,490	\$491,000

Only when the accuracy of the predictor drops to fifty percent (as would be expected for a random predictor) does it become more profitable to take two boxes.

Thus, the decision to take one or two boxes boils down to what the more important issue is for *you*: whether the million dollars is in the opaque box, or whether the predictor is right. Two-boxer focus on the fact that the decision about whether the million is present has already been made before the game begins. You cannot change what is in the boxes; the money's already there, so it's better to take both boxes. One-boxers, on the other hand, insist that, since the predictor is 95% accurate, it is better to play the odds and choose one box, since 95% of people who choose one box leave with a million dollars (and only 5% of those that take two boxes leave with the million and a thousand).

So, one box or two? Perhaps the rational decision is less obvious now. You can't change what's in the boxes, so you should take both boxes. But 95% of the time you'll only get the thousand dollars, whereas if you take just one box, you'll almost certainly walk away with a million. In which case you should have taken both boxes for those extra thousand dollars. Perhaps you'd like to phone a friend?

(Note: for more gullible readers, Eddie McGuire is not really going to host a new game show as described above. The scenario outlined here is actually a version of a well-known paradox, Newcomb's paradox, which is covered extensively in philosophical literature. Not even philosophers can agree as to which is the correct decision to make. Some claim that the very fact that there is no obvious rational decision proves that an accurate predictor of human behaviour cannot exist. Interested readers should check out the works of philosophers David Lewis and Isaac Levi.)

— Graham Waters

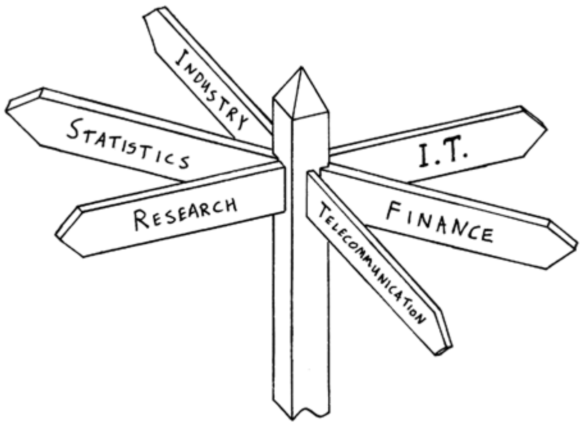
Careers in maths

"High demand for physicists, mathematicians, computer science graduates and bio-scientists is creating unprecedented job and wealth opportunities for young scientists. And many of the best and brightest graduates are being lured overseas, bypassing the Australian workforce altogether." — `mycareer.com.au`

"For the first time in two decades there's starting to be an interest in talented scientists. There's a blossoming of sciences that hasn't existed for a long time." — John Egan, executive pay analyst of Egan Associates.

"... if possible do Maths ... [it] is the single most useful ability to have in your kit-bag to equip you for any eventuality. In any case employers set a lot of store by mathematical ability and are more likely to hire someone with a good background in Mathematics." — Ross Gittens, Economics Editor, *S.M.H.*

"... watch the brokers in suits combing university campuses, desperately trying to find bright, energetic new candidates to fill their dealing desks and research teams. Top dollar is on offer for numerically brilliant, visionary minds ..." — Leonie Wood, *The Age*.



JOB SITES ON THE INTERNET FOR MATHEMATICS

Information about companies which hire mathematicians and job project data:

`www.latrobe.edu.au/www/mathstats/careers.html`

The Mathematical Association of America, Career Profiles:

`www.maa.org/careers/index.html`

Women and Mathematics Network Careers:

`www.mystery.com/~vgkasten/WAM/resources/careers.html`

Dr. Antoinette Tordesillas: (pictured left)

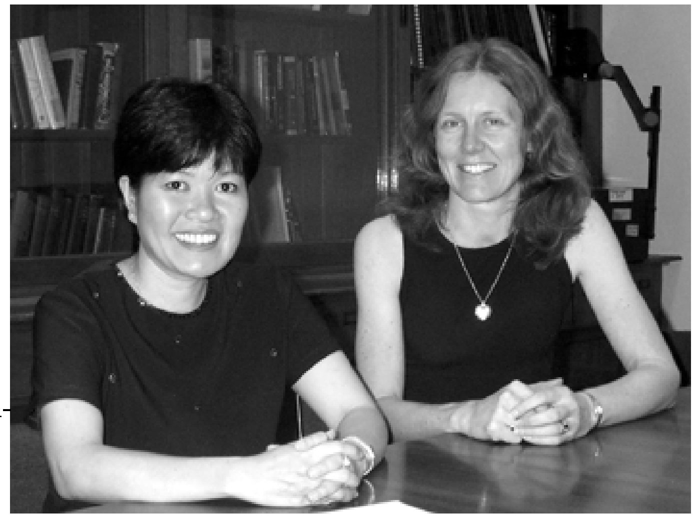
A lecturer in the Department of Mathematics and Statistics at the University of Melbourne, Dr. Antoinette Tordesillas recently received the Australia and New Zealand Industrial and Applied Mathematics (ANZIAM) 2000 J H Michell Medal which recognises "distinguished research by an outstanding mathematician under 35". Dr. Tordesillas received the award during ANZIAM's 36th Applied Mathematics Conference at the Bay of Islands in New Zealand.

Dr. Tordesillas' current research projects include the following, all of which have international links:

1. A USA Army funded project on sand/soil - tyre interactions which has attracted the interest of Australian and Overseas Road Authorities. Her findings could help reduce costly prototype vehicle field testing, provide a more accurate 'feel' for trainee drivers in virtual reality simulators and lead to 'greener' guidelines for off-road driving.

2. A study of the Mechanics of Powders, Sand and other Granular Media, for which Dr. Tordesillas was recently awarded a 2000 Australian Academy of Science grant to allow her to conduct part of this research in the USA next year. The goal of this research is a reliable mathematical model that will run well on readily accessible computers.

3. A study of Insect Olfaction funded by a US National Science Foundation grant in which Dr. Tordesillas is working in conjunction with an American entomologist to understand the ability of insects to sense vanishingly small traces of chemicals in the environment. This work could lead to more accurate prediction of insect behaviour under particular conditions, and to the development of microscale chemical sensors for small robotic devices and other specialised research and industrial applications.



In addition, Dr. Tordesillas was an invited contributor to the panel discussion on "How to Face the 21st Century?" at the 13th International Conference of the International Society For Terrain Vehicle Systems (ISTVS) held last September in Munich, Germany.

Dr. Christine Mangelsdorf:

A lecturer in the Department of Mathematics and Statistics at the University of Melbourne, Dr. Christine Mangelsdorf has just recently been awarded the 1999 Dean's Award for Excellence in Teaching in the Faculty of Science.

Interested in all branches of maths and chemistry, Dr. Mangelsdorf majored in pure and applied maths, physical, inorganic and organic chemistry. Deciding to find an area where she could use her maths and chemistry skills, she pursued an Honours degree and a Ph.D. in applied mathematics with a theoretical chemistry research project. Dr. Mangelsdorf's principal research interest for the last 11 years has been in the area of Colloidal Electrokinetics. This research concentrates on the theoretical modelling of the behaviour of colloidal particles in static and alternating electric fields. The theory and computer programs she has developed are used worldwide by experimentalists.

Over the last 3 years, Dr. Mangelsdorf has also been involved in the scheduling of draws and match programs for various sporting organisations such as the AFL and SANFL.

Since her time as an Honours Student, Dr. Mangelsdorf has been actively involved in promoting mathematics and organising mathematical events, serving as a committee member of MUMS for 7 years and President of MUMS for 1 year.

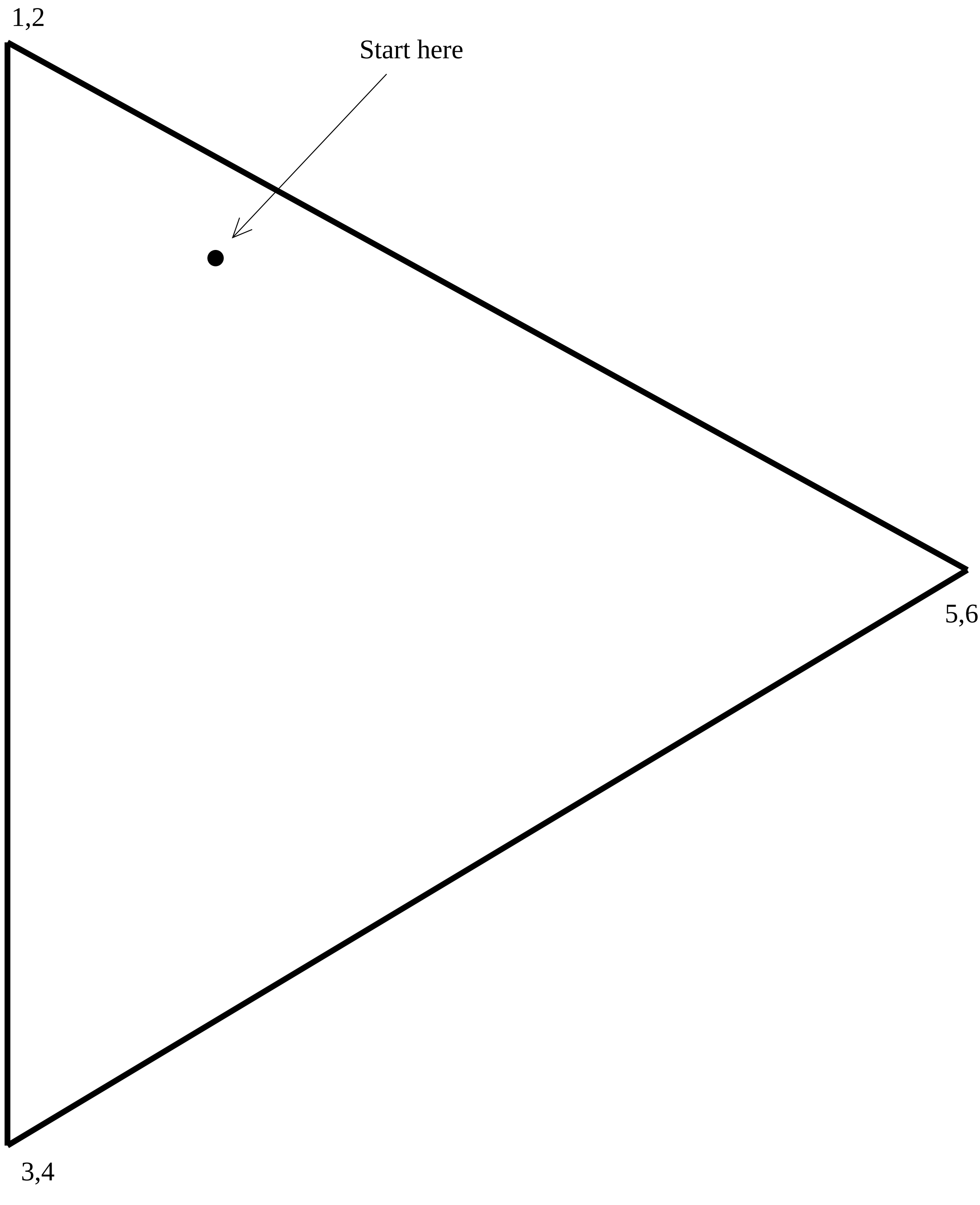
— Sam Richards

Maths Jokes

A professor of mathematics noticed that his kitchen sink had broken down. He called a plumber. The plumber came on the next day, sealed a few screws, and everything was working as before. The professor was delighted. When the plumber gave him the bill a minute later, however, he was shocked. 'This is one third of my monthly salary!' he cried. The plumber said to him, 'I understand your position as a professor. Why don't you come to our company and apply for a position as a plumber? You will earn three times as much as a professor. But remember, when you apply, tell them that you have only completed seventh grade. They don't like educated people.' So the professor got a job as a plumber and his life improved significantly. He just had to seal a screw or two occasionally, and his salary went up drastically. One day, the board of the plumbing company decided that every plumber had to go to evening classes to complete the eight grade. So the professor had to go there too. It so happened that the first class was maths. The evening teacher, to check her students' knowledge, asked for the formula for the area of the circle. The person asked was the professor. He jumped to the board, and then he realised that he had forgotten the formula. He started to reason it, and filled the white board with integrals, differentials and other advanced formulas to conclude the result that he had forgotten. As a result, he got $-\pi r^2$. He didn't like the minus, so he started all over again. He got the minus again. No matter how many times he tried, he always got a minus. He was frustrated. He looked nervously at the class and saw all the plumbers whispering, 'Switch the limits of the integral!'

Q: Why did the chicken cross the Mobius Strip?
A: To get to the same side.

The *Paradox* Sierpinski Gasket Kit



You will need a pen, a ruler and a die.

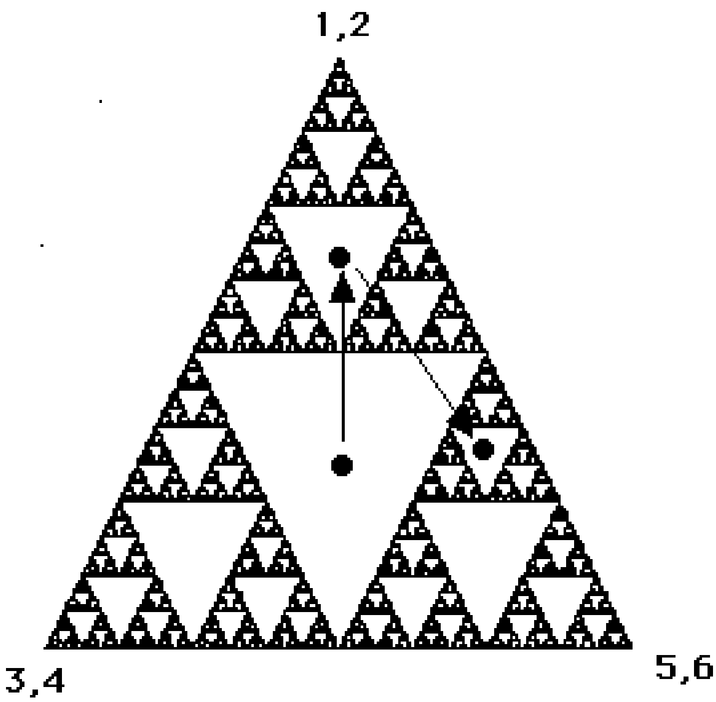
Step 1: Randomly select a vertex of the triangle by rolling the die.

Step 2: Use a ruler to find the midpoint between the dot on the opposite page and the vertex you chose.

Step 3: Make a dot there and use this new dot in step 2 of the next iteration.

Step 4: Go to step 1.

After drawing quite a few dots this way, a fractal similar to the one below will emerge.



www.apesma.asn.au

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2. Work Experience

Where students seek work experience in their chosen field, APESMA provides assistance with information on salaries and conditions of employment.

3. Student Magazine

Student members automatically receive our student magazine, containing articles on where the jobs are, what employers are looking for, work experience and advice from recent graduates.

4. Graduate Salaries

As the official representative of engineers and scientists, APESMA has established award minimums to protect graduate salaries and conditions. We also conduct regular salary surveys to keep graduates up-to-date with market salaries for engineers and scientists. Check out our website for the interactive salary calculator.

5. Contract Advice

When looking for graduate employment, APESMA provides confidential advice on the salary and conditions of your job offer. This advice ensures new graduates make the most of their professional employment.

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Students can contact APESMA for individual advice or assistance on employment-related matters, as required.

7. Member Services

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8. Club Funding

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Bad proofs

Several years ago, Assoc. Prof. Barry Hughes set a first-year exam question which asked for a proof that there is no largest prime number. The following memorandum was sent to him by one of the graduate students who helped with the marking.

Hi Barry,

Just for educational value, I compiled a list of the most incorrect answers to question B4(b). It is entitled "15 good reasons why Pure Mathematics is not taught to first year students."

1. Proof by example
   "Let x be the largest prime. Then $x = 91$ but $91 + 6 = 97$ which is prime. Therefore 91 cannot be the largest prime number. Therefore there is no largest prime number."

2. Proof by oddness
   "If n is the largest prime number, then n is odd. Then $(n+1)/2$ is even. Therefore $(n+1)/2 + n$ is odd. But $(n+1)/2 + n$ is not divisible by any number except itself. As it is bigger than n, the assumption is wrong, by contradiction."

3. Proof by intuition
   "Prime numbers are integers that can be divided by themselves only; prime numbers are odd with the exception of 2. By intuition as $n \to \infty$, there will always be an odd number that cannot be divided by any other number besides itself."

4. Proof by $\sqrt{2}$
   "Assume that p is divisible by q, i.e. $p/q = 2r$ where r is an even number. Then $p = 2qr$ so $p^2 = 4q^2r^2$. But r^2 does not exist and $q! = 1$. Therefore q must exist. Since q exists, p must be divisible. Therefore, by contrary, there is no largest prime number."

5. Proof by superinduction
   "2 is a prime number. Now assume N is the largest prime. But then $N + 1$ exists and is also prime. Therefore by induction there is no highest prime number."

6. Proof by the previous question
   "Suppose N is the largest prime. Then let $N = (n^2)/2$. Therefore $n = \sqrt{2N}$. But from above $1 + 2 + 3 + ... + n > N$. Hence there is a larger prime number than N."

7. Proof by tutorial question
   "Let m, n be two integers with $m > n + 1$. If k is even, $m^k + n^k$ cannot be expressed in terms of $(m + n)$(polynomial in m and n) and so is prime. Therefore as m and n can be any numbers, there is obviously no largest prime number."

8. Proof by having no idea what a prime is
   "Say the largest prime possible is x, then $2x$ is also a prime since the statement is true for all natural numbers."

9. Proof by experimental data
   "Suppose n is the highest prime. Then $2n-1$ is also prime. But $2n-1 > n$ so there is no highest prime. (Check: $2 \times 2 - 1 = 3$, $2 \times 3 - 1 = 5$, $2 \times 5 - 1 = 11$, $2 \times 11 - 1 = 23$, so true)"

10. Proof by subscript
    "If there is a highest prime, we can number all the primes $p_1, p_2, \ldots, p_n$. But as there is no highest natural number, there is always an $n+1$ so there must be a p_{n+1}. Therefore there is no highest prime."

11. Proof by infinity
    "Let n be the highest prime number. But ∞ is greater than all numbers so $\infty > n$. If n is the highest prime this would mean ∞ has factors. Therefore we have a contradiction."

12. Proof by reverse logic
    "All prime numbers are odd. Suppose there were a highest prime. Then we have a highest odd number. But if $2k+1$ is the highest odd number then $2k+3 = 2(k+1)+1$ is also an odd number. Therfore we have contradiction and therefore we have a contradiction."

13. Proof by denial
    "Assume there is a largest prime M. We can add 1 to M until we get another prime number N $(M + 1 + 1 + 1 + ... + 1 = N)$. But then $N > M$. Therefore M is not the largest prime number, so there is no largest prime number."

14. Proof by formula
    "As prime numbers are derived via the formula, we can assume it works for $n = k$ giving the highest prime number. But then it also works for $n = k+1$, so there is no highest prime number."

15. Proof by continuity
    "Let x be the largest prime number. Then $x >$ all other primes. But then $(x+n)$, the next prime number, does not exist. However numbers are continuous and so $(x+n)$ does exist. Therefore there is no x."

These are all verbatim answers from the 150-odd papers I marked, i.e. about one in ten. My favourite is definitely number 10, I think that is quite ingenious.

Who would rather be a billionare?

THE BILLIONIST MANIFESTO

In the beginning was the word, and the word was "billion". Mathematicians, the guardians of logic, bastions of academic rigour, defenders of simplicity, wardens against pragmatism, stalwarts of beautiful propositions and masters of numbers (before they moved on to more interesting things), are generally efficient with notation and nomenclature. In typical logical and elegant fashion, noticing they usually had ten fingers, they designed an ingenious base 10 numbering system. They named 10 ten, 100 one hundred, 1,000 one thousand, and 1,000,000 one million. This system worked well for most everyday calculations, and could even be used tastefully and without repetition for larger calculations, to talk about larger powers of 10: for example, 10,000,000 was ten million and 1,000,000,000 was one thousand million - a simple, obvious but nonetheless aesthetically pleasing construction.

However, when they came to 10^{12} the mathematicians ran into a problem. Without any new words, 1,000,000,000,000 must be described as one million million or one thousand thousand million or one ten hundred thousand million, all of which are unsightly, repetitive, inefficient and prone to confusion. Thus, the good mathematicians deemed this number a "billion" - an obvious simplification to avoid awkward descriptions of large numbers.

And so the human race lived in harmony with the universe for many years, with a happy people and a simple number system. Unfortunately then capitalism and its associated greed and corruption came along, and a few people started getting extremely rich. They gained more and more money, but were never happy with their hollow material fortunes, and constantly pursued greater quantities. These few insidious, depraved individuals had nothing better to do so they counted the number of monetary units they possessed. First they gained a hundred monetary units, then a thousand, then ten thousand, one hundred thousand, and then passed a point where a certain elite few owned over *one million* monetary units.

As any mathematician can remark saliently, this is not a particularly noteworthy milestone since one million is only a special number in a base ten numbering system, ten being the completely arbitrary number of fingers human beings have on our hands and human beings being a completely arbitrary and mostly flawed quirk of nature. Moreover, the value of the monetary unit was completely arbitrary and not even constant due to the idiotic humans' inability to figure out a stable economic system. Even in a base 10 numbering system, it's still not very interesting - far more profound are numbers like 239[*], 561[†], 1729[‡], 17163[§], or 357686312646216567629137[¶].

[*] The largest integer which cannot be written as a sum of fewer than 8 perfect cubes.

[†] The smallest positive composite integer n such that $a^{n-1} - 1$ is divisible by n for every integer a relatively prime to n.

[‡] The smallest positive integer expressible as a sum of two cubes in two different ways (with apologies to Hardy and Ramanujan).

[§] The largest integer which is not the sum of distinct squares of prime numbers.

[¶] The largest left truncatable prime in base 10 - whenever a group of leftmost digits are removed, the number remains a prime.

However uninspiring such an achievement was to the wise mathematicians, to the hoarders of wealth it was most extraordinary, since they could now give themselves an unprecedented ego boost by calling themselves "millionaires". The millionaires were not happy, however, because their only friends were other unhappy millionaires, and decided that the only way to rectify the situation was to get more money.

Thus the world was plunged into a dark era of scrambles for wealth by rich bastards. The masses became cynical and disillusioned, and the upper class became even less content, despite their troves growing in geometric proportions. In no time at all their stocks bulged from one million to one thousand million units.

The capitalistic overlords then sensed it was time for a new catchphrase for the masses to utter in their worship, but they were, like many, ignorant. They believed that they had reached a new level in their superiority since their hoardings of arbitrary monetary units had gained another three arbitrary zeros. The next major numerical unit, they thought, was the "billion", and the marketing executives deemed "thousand-millionaire" unmarketable due to its awkward wording making it inaccessible to the masses, who were now renamed the "market". Renaming themselves "billionaires", the rich bastards imposed further idolistic worship from the lower classes of society. The impoverished and oppressed mathematicians were passed by the wayside despite their final stand for truth and integrity, and the physicists, who had naively been entrusted by the mathematicians with the safekeeping of arithmetic notation while the mathematicians moved on to more interesting matters, were exuberantly trampled. Thus a powerful, insidious few rewrote history and with their propaganda forced doublethink on the masses with regard to this precious word. Through a saturation advertising campaign they deeply ingrained into the common people's collective consciousness that a billion had always been and would always be one thousand million or 10^9. The mass media under their tyrannical control was unrelenting in its bombardment of this unjust, inaccurate and aesthetically inferior misinformation on the unfortunate, innocent and undefended viewing public.

And now, we live in a time where it is unthinkingly accepted by almost all that a billion is 1,000,000,000, and a billion (10^{12}) is now grotesquely renamed a "trillion". This cannot go on! Let not the rich bastards dictate our mathematical notation! They can take our arbitrary monetary units, they can take our arbitrary zeroes, they can even take our seats at Colonial stadium, but they cannot undermine our proper nomenclature! Let them fill our minds with Popstars and take away our Hey Hey It's Saturday, but let them not redefine what was a perfect aesthetic numbering system! This time they have gone too far. We shall not reinvent ourselves just to appease some over-inflated egos. A billion must remain a billion.

BILLIONISTS OF THE WORLD UNITE!

— D.V. Mathews

Solutions to last issue's problems

Problem 1: Do there exist irrational numbers a and b such that a^b is rational?
Solution: First, we note that $\sqrt{2}$ is irrational. Now let $a = \sqrt{2}$ and $b = \sqrt{2}$. We have two options.

1. If a^b is rational, then we have a solution. Otherwise, a^b is irrational, but then
2. let $a = \sqrt{2}^{\sqrt{2}}$ and $b = \sqrt{2}$. Then $a^b = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^2 = 2$ which is rational.

Hence, there do exist irrational numbers a and b such that a^b is rational.

Problem 2: The sequence of positive integers $a_1, a_2, \ldots$ is such that $a_{a_n} + a_n = 2n$ for all n. Find all such sequences.
Solution: If $a_m = a_n$ then $a_{a_m} = a_{a_n}$ and by the condition, $a_{a_m} + a_m = a_{a_n} + a_n$. Thus, m = n and we conclude that all members of the sequence are distinct. For $n = 1$, $a_{a_1} + a_1 = 2$ so $a_1 = 1$. We shall prove that $a_n = n$ for all n by induction. Now suppose $a_i = i$ for $i = 1, \ldots, n-1$. Now a_n is at least n by the fact that all members of the sequence are distinct. Similarly, a_{a_n} is also at least n but their sum is $2n$. Hence, $a_n = a_{a_n} = n$, as required. Clearly, this sequence satisifies the conditions of the question and is the only such sequence.

Problem 3: Two grasshoppers are sitting at the endpoints of the interval $[0, 1]$. A set of n points of the segment are marked, dividing $[0, 1]$ into $(n + 1)$ intervals. A grasshopper can choose any of the marked points and jump over it to the point symmetric to his previous location, provided that this symmetric point also belongs to the segment $[0, 1]$. For one move, the grasshoppers either jump simultaneously according to this rule, or else one of them jumps and the other stays where it is. What is the least possible number of moves needed to ensure that the grasshoppers occupy locations in the same interval (that is, with no marked points between them)?
Solution: It is easy to verify that if the points 9/23, 17/23 and 19/23 are marked, it is impossible for the grasshoppers to jump into the same interval in one move. Let us prove that they always can jump into the largest interval in two moves. It is sufficient to prove this only for the grasshopper starting at 0 since the same proof will be valid for the other grasshopper as well. Let a be the left end of the interval $[a, a + s]$ which has the largest length s. (If there is more than one interval of length s, then pick an arbitrary one.) If $a < s$, the grasshopper can jump into this interval in one move, jumping from 0 to $2a$. When $a \geq s$, consider the segment $[(a - s)/2, (a + s)/2]$ of length s. Due to the maximality of s, this segment contains at least one marked point b. Jumping from 0 to $2b$, the grasshopper will appear in the interval $[a - s, a + s]$. If this point is still not in the interval $[a, a + s]$, the second jump over the point a will take it into the interval $[a, a + s]$ as required.

Problems

The following are some maths problems for which prize money is offered. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number. Solutions may be emailed to `paradox@ms.unimelb.edu.au` or you can drop a hard copy of your solution into the MUMS pigeon-hole near the Maths and Stats Office in the Richard Berry Building.

1. (\$5) Can a surgeon with only two pairs of sterile gloves perform operations on three distinct patients and keep everybody safe?

2. (\$5) Back in the good old days when one and two cent pieces were abundant, George's attention was caught by a big neon sign on the supermarket which simply read "7.11". Puzzled, he entered the shop and found himself speaking to the shopkeeper.

   George: What does it mean?
   Shopkeeper: Oh, it's simple. We are open from 7am till 11pm.

   Indeed it was simple. But George, not satisfied with his newfound knowledge, decided to purchase four products from the colourful shelves as well.

   Shopkeeper: That'll be 7.11, thanks.
   George: What? Why?
   Shopkeeper: It's seven dollars and eleven cents for all the goods you have chosen, sir.
   George: Because of the opening hours?
   Shopkeeper: No, sir. I have just taken note of the prices of the goods, then I have multiplied them and the result is 7.11.
   George: You did what?! You should have added them, not multiplied!
   Shopkeeper: Indeed, sir, I must apologise — you pay 7.11.
   George: You've got to be joking!
   Shopkeeper: No, I've added them this time. Check it yourself.

   After a thorough verification George had to admit that the cashier, or rather his computer had made no mistake in either calculation. What were the prices of the goods acquired by George?

3. (\$10) Let ABC be an isosceles triangle with $AB = AC$. Suppose that the angle bisector of $\angle ABC$ meets AC at D and that $BC = BD + AD$. Determine $\angle BAC$.

4. (\$10) Does there exist an infinite sequence of 0's and 1's in which no finite block of any length occurs three times in succession?

— Norman Do

The *Paradox* team would like to thank Charles Gutjahr, Dougal Ure, Margaret Shaw and Chaitanya Rao for assisting with this issue.

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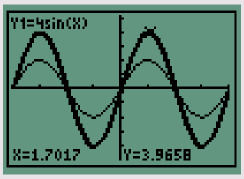

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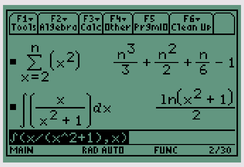

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